Exercise 3.4 Let be a direction (i.e., unit 3-vector) in spherical coordinates. Compute the eigenvalues and eigenvectors for .

Solution.

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To find the eigenvalues we solve the characteristic equation:



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To find the eigenvectors we assume  and then solve the eigenvector equation :





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For ** = 1:





For ** = -1:

 





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Note 1: Multiplying either  yields an equivalent solution.

Note 2: Had we assumed  as we did in Exercise 3.3, without the phase factor, that would have been equivalent to setting ** = 0. That would have meant that  and so we could not have obtained a solution for general .